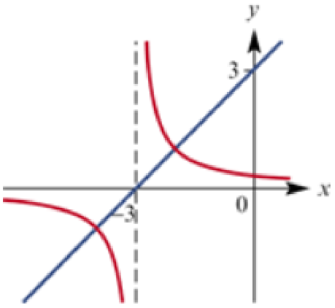
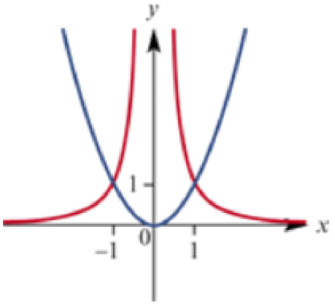
Exercise 12A: Solutions

1 a



b



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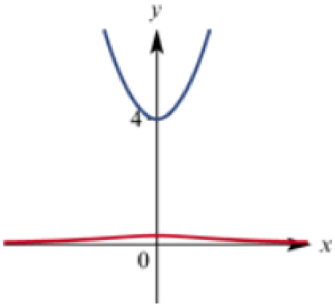

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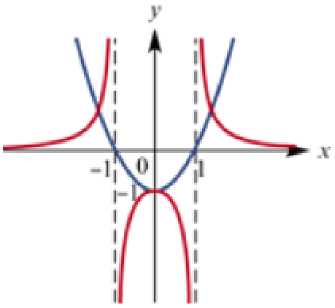

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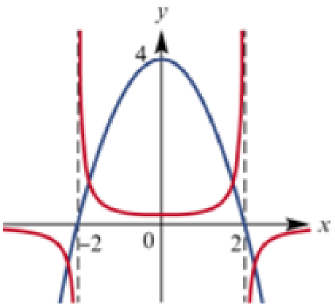

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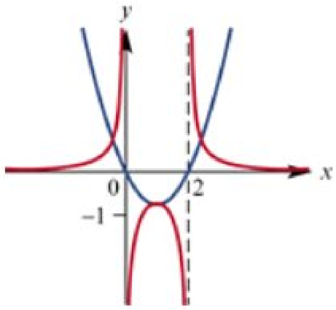

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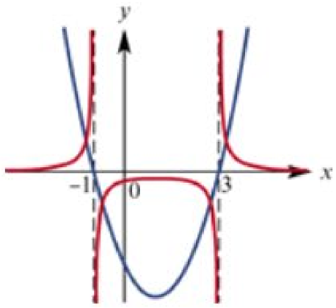

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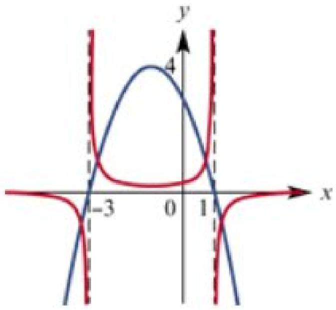
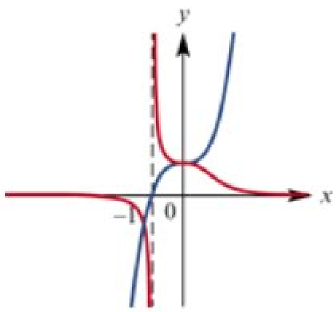
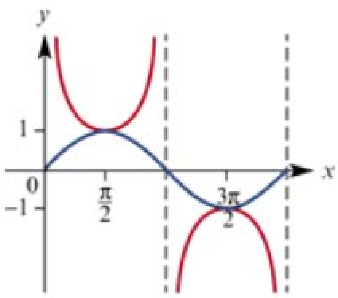


i



2 a



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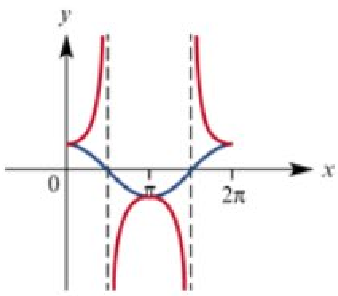

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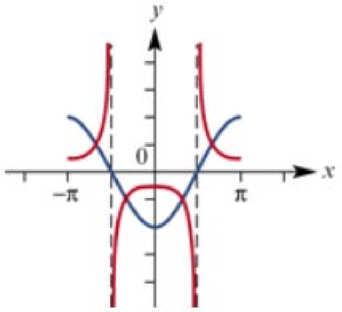
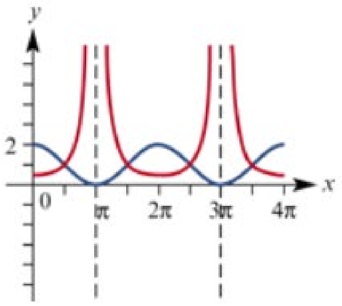


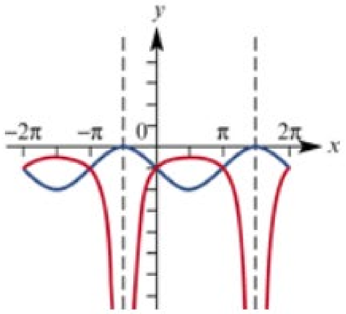
y
x
0
−π
π

d



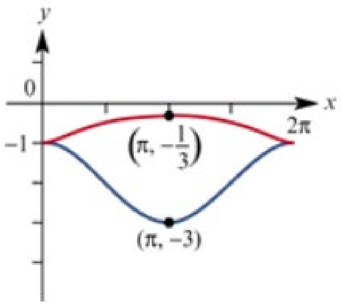
y
x
2
0
π
2π
3π
4π

e



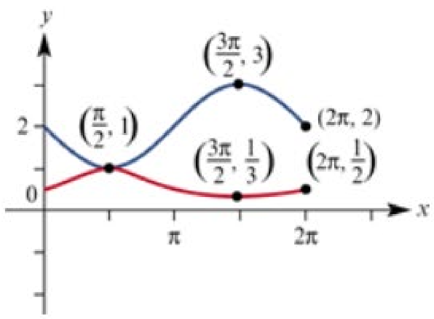
y
x
−2π
−π
0
π
2π

f



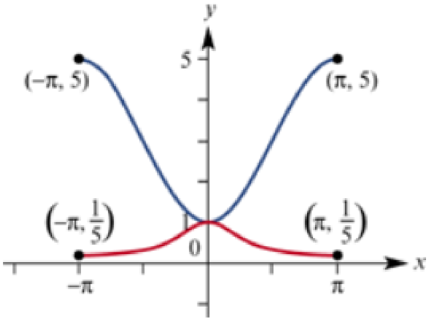
y
x
0
−1
2π
(π, −1/3)
(π, −3)

g



y
x
2
0
π
2π
(3π/2, 3)
(π/2, 1)
(2π, 2)
(3π/2, 1/3)
(2π, 1/2)

h

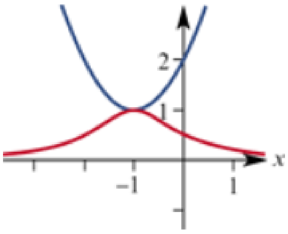


3 a We complete the square so that

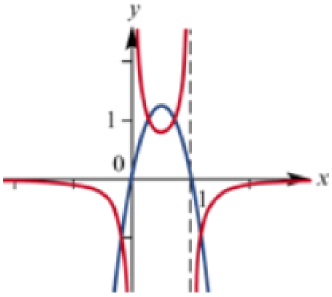
$$\begin{aligned} f(x) &= x^2+2x+2 \\ &= (x^2+2x+1)-1+2 \\ &= (x+1)^2+1. \end{aligned}$$

Therefore, a minimum turning point is located at point $(-1,1)$.

b



4 a



b To find points of intersection we solve two equations: $f(x)=1$ and $f(x)=-1$. If $f(x)=1$ then

$$5x(1-x)=1.$$

Solving this quadratic equation (using the quadratic equation or your calculator) gives

$$x=\frac{5\pm\sqrt{5}}{10}.$$

Since $f(x)=1$, the coordinates are

$$\left(\frac{5\pm\sqrt{5}}{10},1\right).$$

If $f(x)=1$ then

$$5x(1-x)=-1.$$

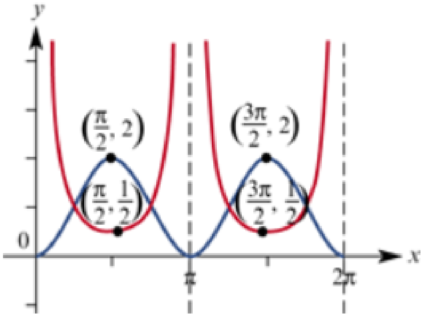
Solving this quadratic equation gives

$$x=\frac{5\pm 3\sqrt{5}}{10}.$$

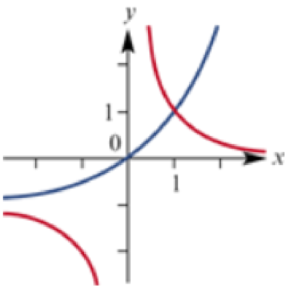
Since $f(x)=-1$, the coordinates are

$$\left(\frac{5\pm 3\sqrt{5}}{10},-1\right).$$

5 Notice that $y = 2\sin^2 x$ will have the same x-intercepts as $y = 2\sin x$ but will be non-negative for all values of x.



6



7 a We complete the square so that

$$\begin{aligned} f(x) &= x^2 + 2kx + 1 \\ &= (x^2 + 2x + k^2) - k^2 + 1 \\ &= (x+k)^2 + 1 - k^2. \end{aligned}$$

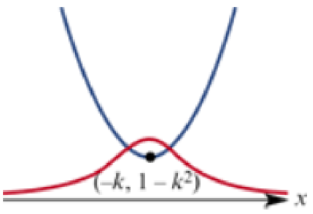
Therefore, a minimum turning point is located at point $(-k, 1-k^2)$.

b i The graph of $y = f(x)$ will have no x-intercept provided $1 - k^2 > 0$. This means that $-1 < k < 1$.

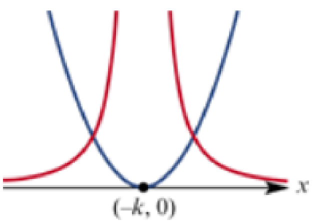
ii The graph of $y = f(x)$ will have one x-intercept provided $1 - k^2 - 0$. This means that $k = \pm 1$.

iii The graph of $y = f(x)$ will have two x-intercepts provided $1 - k^2 < 0$. This means that $k > 1$ or $k < -1$.

c i



ii



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